Is there a need to redefine Mw and Me?

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The realistic assessment of earthquake size and related seismic hazard and risk Requires to consider at least two physically defined source parameters:

- **Seismic moment** **M_0 = μ⊠D** A as a **static measure of EQ size** **(tectonic effect)**
- **Seismic energy**

$$E_S = \left[\frac{2}{15\pi\rho\alpha^5} + \frac{2}{10\pi\rho\beta^5}\right]\int_{f1}^{f2}\left|\frac{\dot{u}(f)}{G(f)/2\pi f}\right|^2 df$$

as a **dynamic measure of EQ strength**

M_0 and E_S are complementary because the ratio M_0/E_S depends on stress drop and rupture velocity which may vary by about 3-4, resp. more than 1 order of magnitude .

Accordingly, for equal M_0 the corner frequency f_c of the source spectrum and the released E_S may vary up to more than 1 order (see Figs 1 and 6).

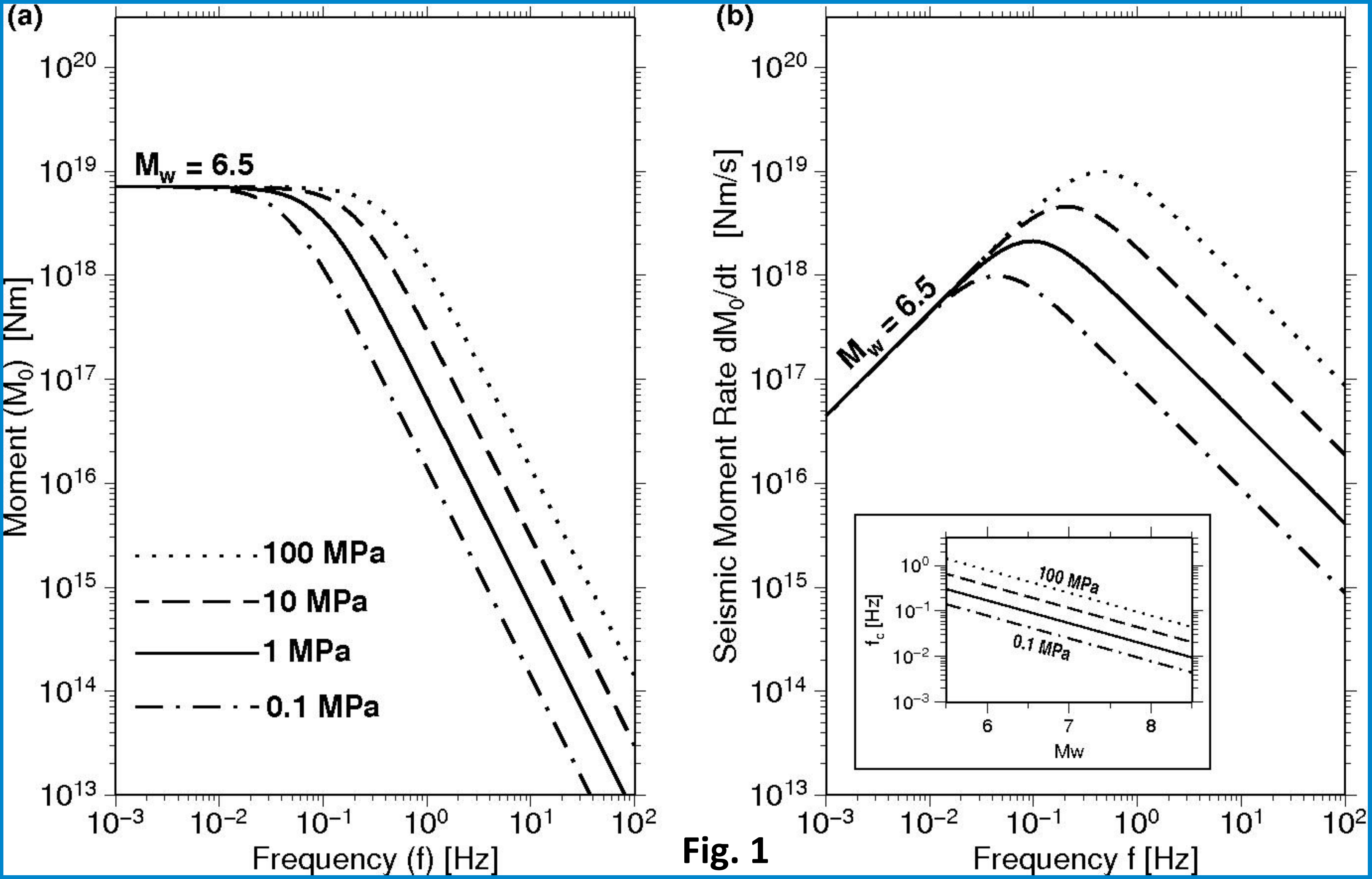


Fig. 1

In order to make M_0 and E_S data handable for practical applications such as rapid EQ size/strength and related hazard/risk potential assessment they have been scaled to EQ magnitude via semi-empirical relationships.

The most fundamental relationship is that of Gutenberg (1956) between broadband body-wave magnitude **mB = m** and seismic energy which reads with E_S in units of Joule:

$$\log E_S = 2.4\, mB - 1.2 \qquad (1)$$

which is, however, based on rather meager data!

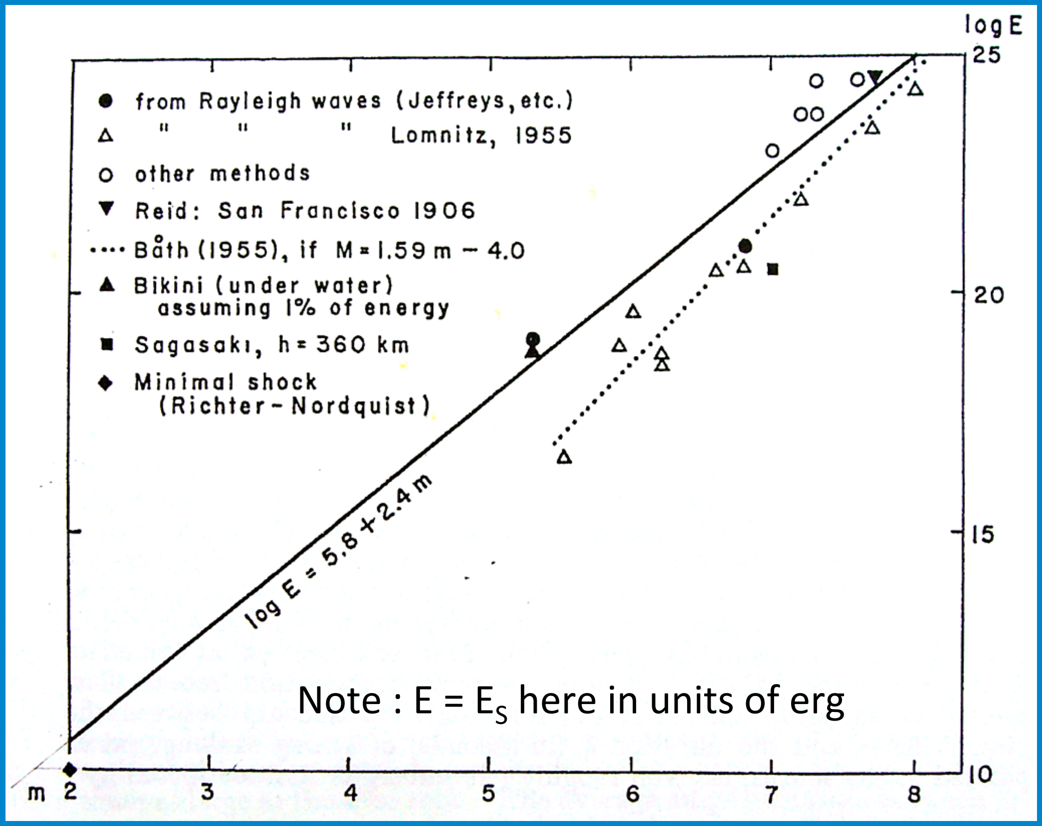


Fig. 2

Fig. 3 illustrates how much mB and thus E_S may vary for equal moment magnitude Mw but different rupture duration and

Fig. 4 how well modern IASPEI broadband mB correlates with E_S but **different from Eq. (1)!**

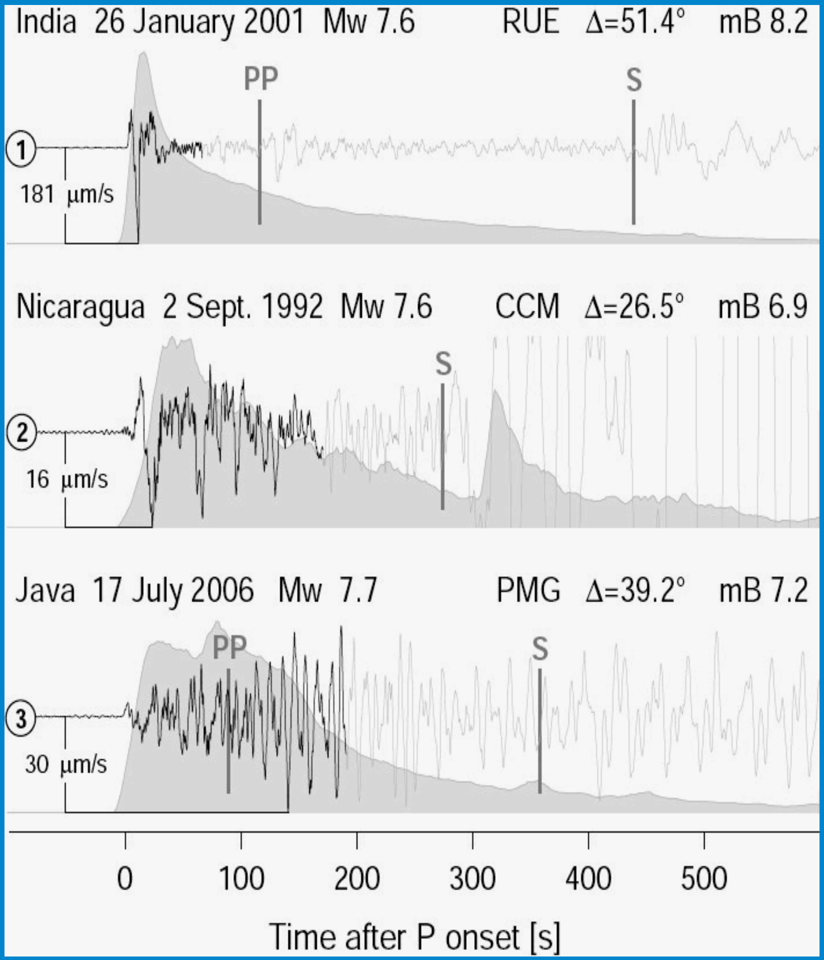


Fig. 3

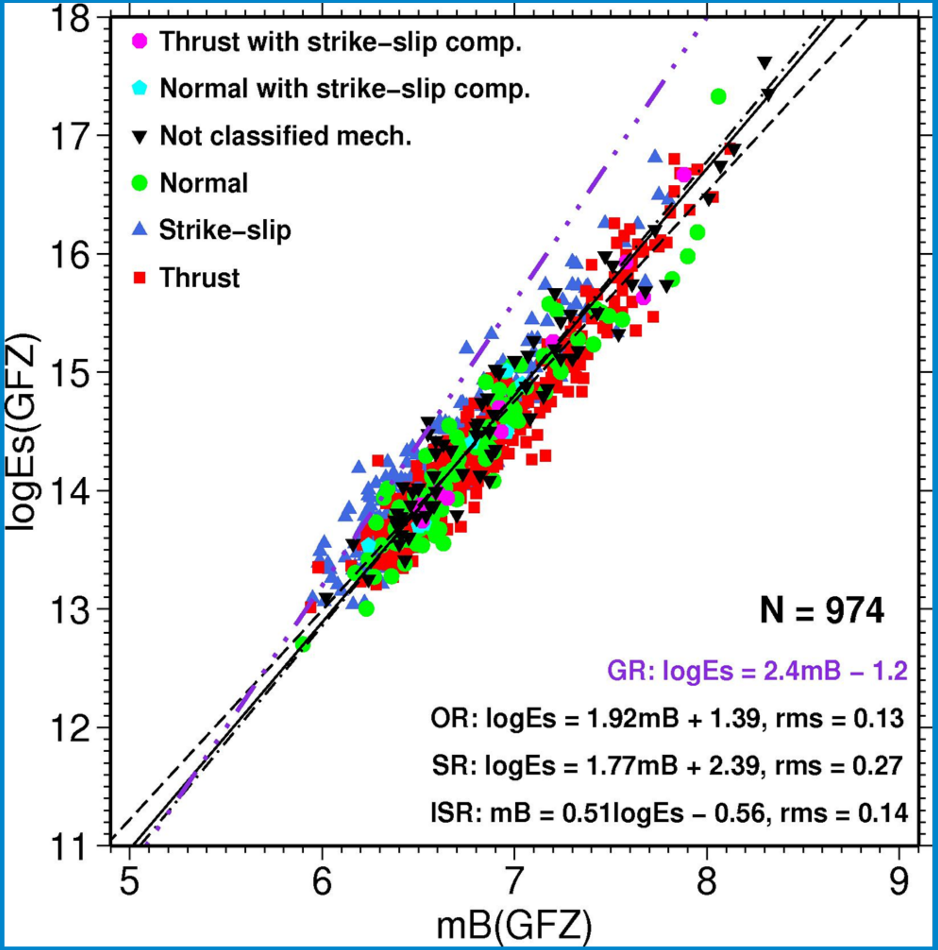


Fig. 4

When Kanamori (1977) became interested in the E_S release of great EQs he could not use Eq. (1) because **with the introduction of the WWSSN no more mB was determined by US agencies and short-period mb heavily underestimated the magnitude of great EQs.**

Since only Ms was available as a reliable magnitude estimate for EQs with $\log M_0 \approx 19\text{-}21$ **Kanamori had to make use of the Richter (1958) relationship**

$$\log E_S = 1.5\ Ms + 4.8 \qquad (2)$$

which follow by inserting into Eq. (1) the other fundamental Gutenberg-Richter (1956) empirical relationship

$$mB = 0.63\ Ms(20) + 2.5 \qquad (3)$$

and resolving it for $\log E_S$.

By assuming a **constant ratio** $E_S/M_0 = 5\times10^{-5}$ or $\mathbf{\Theta = \log (E_S/M_0) = -4.3}$ Kanamori (1977) and later Hanks and Kanamori (1979) derived the moment magnitude formula. It reads in the IASPEI (2013) standard notation

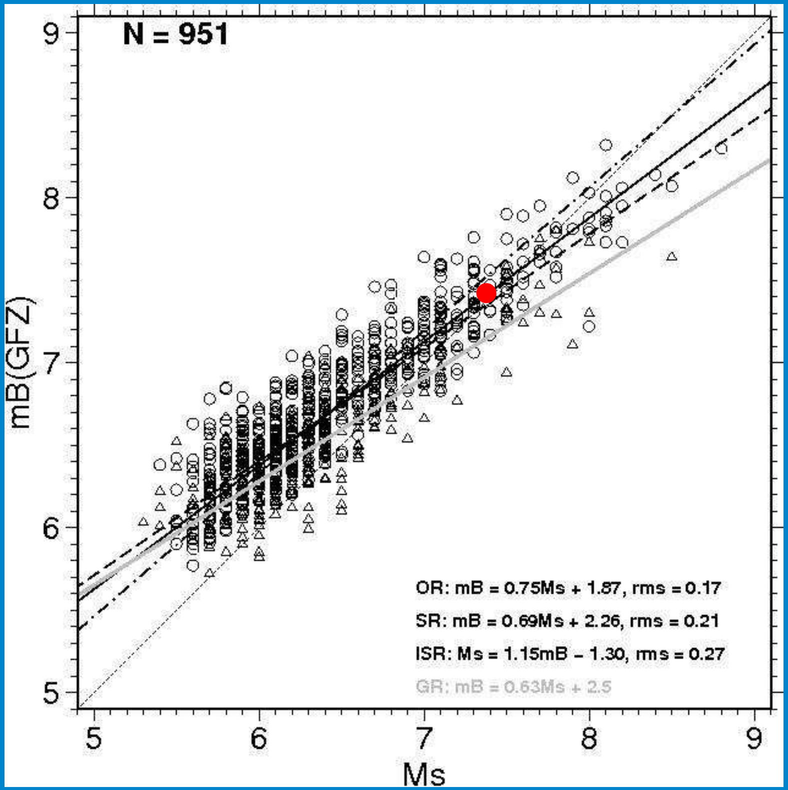
$$Mw = (\log M_0 - 9.1)/1.5. \qquad (4)$$

Is this **Mw formula** still in agreement with modern data?

According to Fig. 4 **log Es = 1.92mB + 1.39 with rms = 0.13** **(5)**

$\neq$ GR **log E_S = 2.4mB – 1.2**

According to Fig. 5



mB_BB = 0.75 Ms(20) + 1.87 **(6)**

$\neq$ mB = 0.63Ms + 2.5 (G-R 1956)

$\Rightarrow$ mB_BB = Ms for 7.48

$\Rightarrow$ $\neq$ mB = Ms for 6.75 (G-R 1956 and Ekström & Dziewonski 1988)

However, when inserting (6) into (5) yields

$\log E_S$ = 1.44Ms + 4.98 $\approx$ $\log E_S$ = 1.5Ms + 4.8 Richter (1958)

At periods around 20 s Ms(20) samples on average the energy-release maximum of the source spectrum of earthquakes with Mw ≈ Ms ≈ 7.5 and broadband mB(BB) as well ⇒ i.e., mB(BB) ≈ Ms = 7.5

At Mw ≥ Ms < 7.5 20 s Ms, however, does on average no longer sample the energy-release maximum and therefore tends to systematically underestimate $\log E_S$, in contrast to mB(BB) which samples in a wide magnitude range down to below M = 5 always the energy release maximum.

This explains, why at Ms ≈ 5.5 mB(BB) is on average ≈ 6 (see Fig. 6).

For G-R traditional mB = Ms at 6.75 because of more limited bandwidth recordst owards Longer periods and thus mB rarely sampled at T > 10-12s.

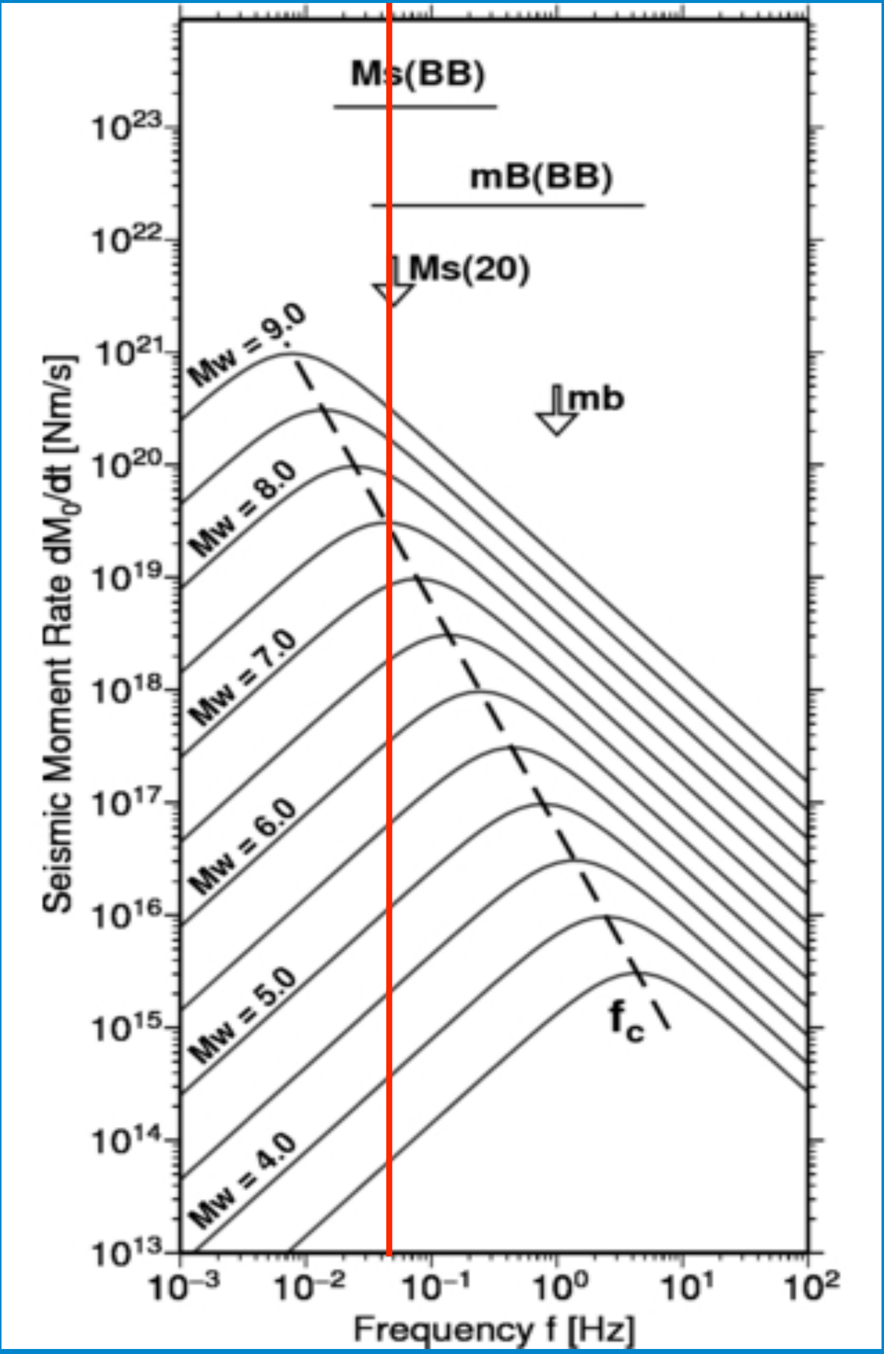
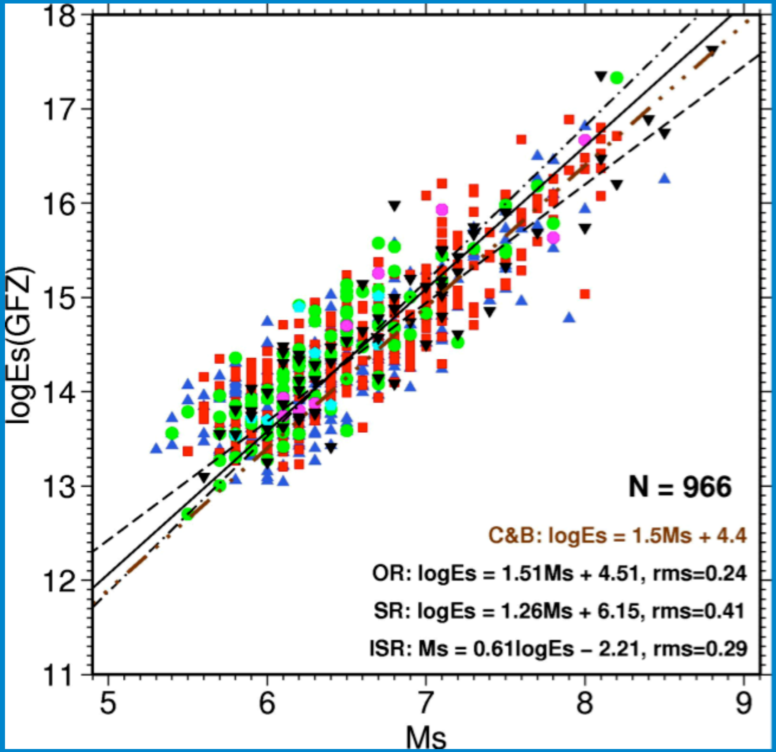


Fig. 6

But we nowadays we do no need the detour via the logEs-mB and mB-Mw relationships

According to Fig. 7



Direct OR regression of GFZ logEs over Ms (NEIC) yields

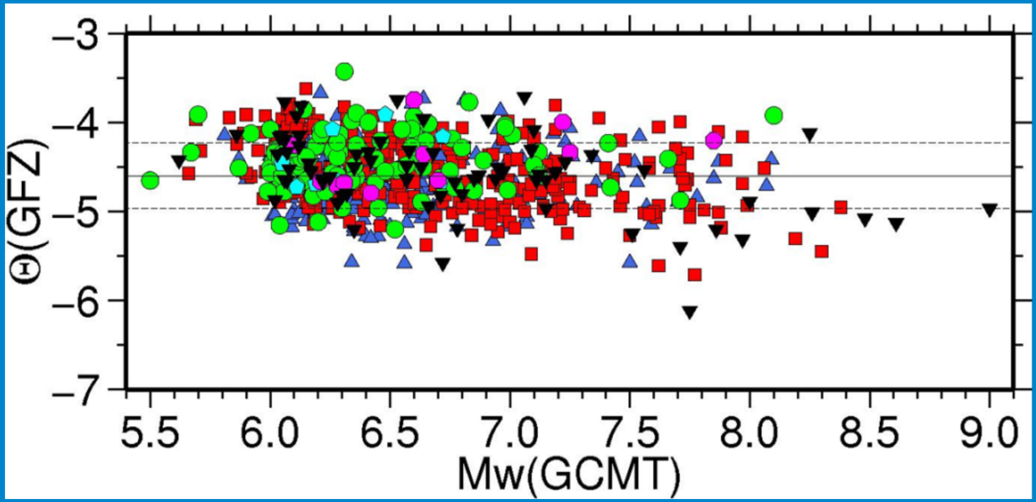
logEs = 1.51Ms + <u>4.51</u> with rms = 0.24 (7)

≠ Richter $\log E_S = 1.5\ Ms + \mathbf{4.8}$

but close to Choy and Boatwright (1995)

$\log E_S = 1.5\ Ms + \mathbf{4.4}$

According to Fig. 8



$\Theta = \log(E_S/M_0) = -4.6$ (8)
GFZ

≠

$\Theta = \log(E_S/M_0) = -4.3$
Kanamori

When deriving a new Mw formula on the basis of eqs. (7) and (8) on gets

$$\mathbf{Mw = (logM_0 - 9.11)/1.51} \approx Mw = (logM_0 - 9.1)/1.5 \qquad (9)$$

with differences between -0.04 and -0.07 m.u. only in the range $logM_0$ = 17 - 23, i.e., the IASPEI standard formula according to Kanamori (1977) is still in reasonably good agreement with the new IASPEI standard magnitudes mB_BB and Ms_20, their interrelationship and their relationship with directly measured GFZ E_S and Θ values.

However, when repeating this procedure with the much more noisy $logE_S$(USGS) data (Figs. 9 and 10) then on gets

$$\mathbf{Mw = (logM_0 - 7.21)/1.79} \neq Mw = (logM_0 - 9.1)/1.5 \qquad (10)$$

with differences ranging between +0.2 and -1.7 m.u. between the new and current formula.

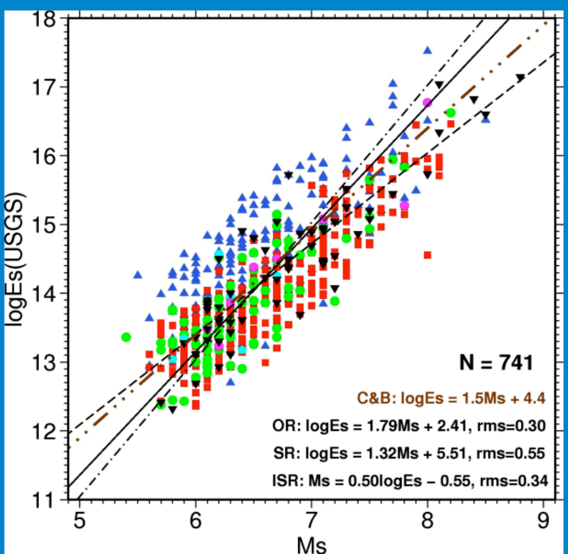


Fig. 9

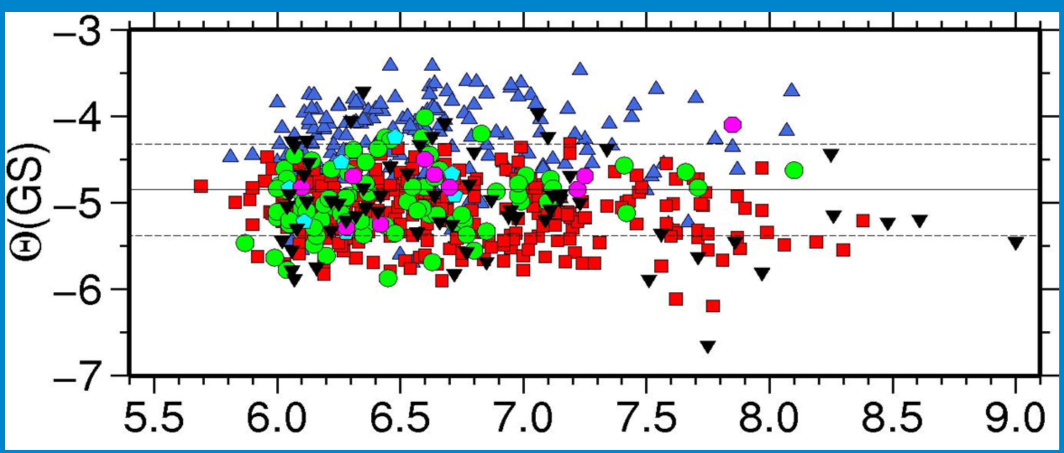


Fig. 10 Θaverage = - 4.8

Is the current **Me formula** still in agreement with modern data?

Choy and Boatwright (1995) scaled their directly measured and model-based calculated broadband E_S values to Ms(NEIC) in order to reproduce Eq. (2)

In a semi-heuristic regression, aimed at assuring best possible continuity with the classical Richter (1958) logEs-Ms formula, they assumed the correctness of the slope of 1.5 and only looked for a better least-square solution of the constant.

Thus they derived the revised relationship

$$\log E_S = 1.5 Ms + 4.4, \qquad (11)$$

substituded in (6) Ms by Me, resolved it for Me and thus arrived at

$$Me = (\log E_S - 4.4)/1.5 \qquad (12)$$

Eq. (11) could be almost perfectly reproduced by orthogonal regressing GFZ $\log E_S$ over Ms(NEIC) ⇒ $Me = (\log E_S - 4.51)/1.51$ **(13)**

however not by orthogonal regression through the much more noisy NEIC $\log E_S$ data over Ms(NEIC) ⇒ $Me = (\log E_S - 2.41)/1.79$ (see Fig. 11) (14)

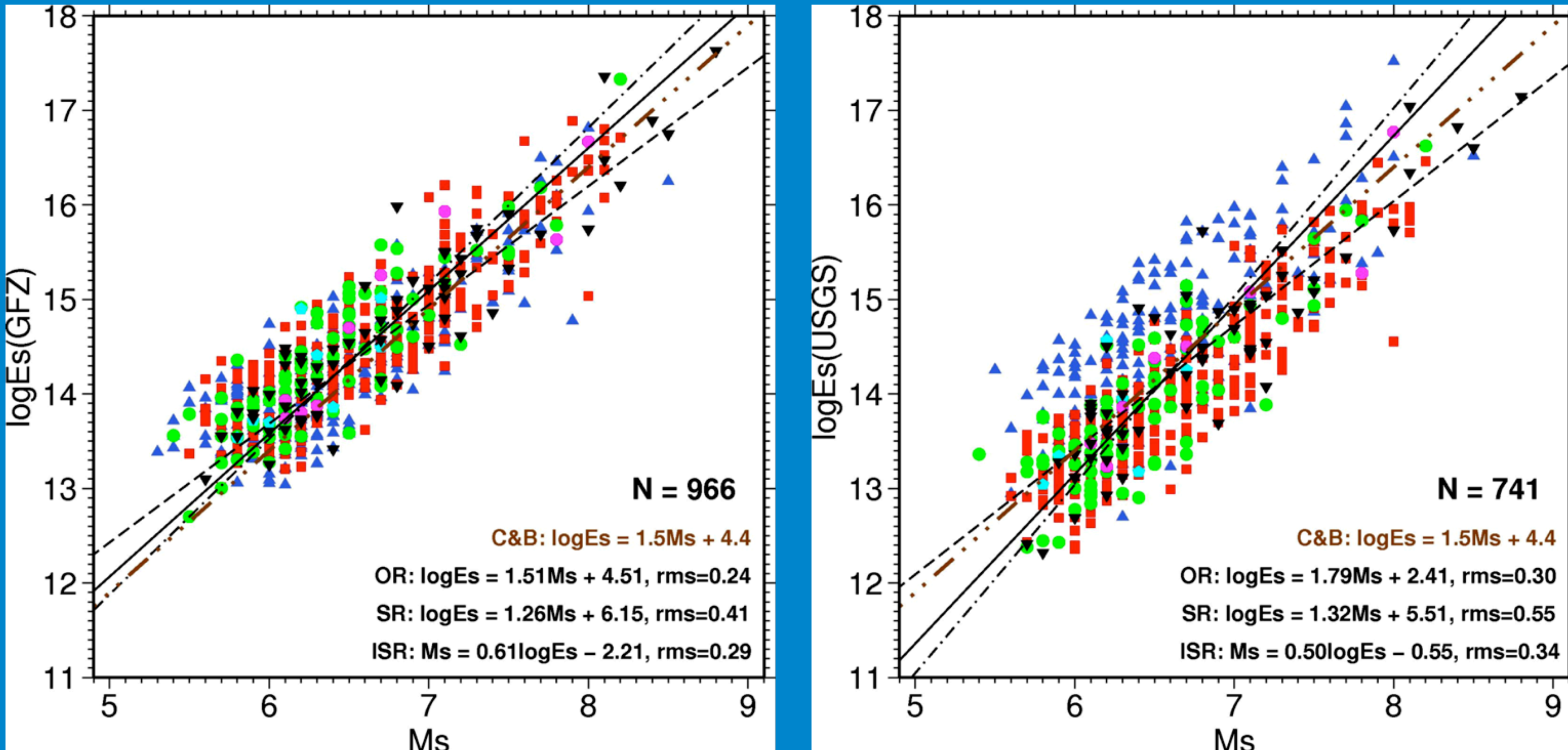


Fig. 11

The new Me formula (13) based on GFZ log E_S data over Ms(NEIC) between 5.4 to 8.8 would yield Me values that are between **-0.10 and -0.14 m.u.** smaller than those according to the current Me standard formula.

In contrast , a new Me formula based on an orthonal regression of NEIC log E_S data in the same range over Ms would yield Me values that differ between **+0.30 and -0.37** m.u. from those derived by the current Me standard formula.

Discussion

The larger scatter of NEIC $\log E_S$ data are mainly due to still much debated, mainly theoretically – under simplified assumptions with respect to the rupture process and wave propagation in a homogeneous 1D and non-scattering medium – derived source-mechanism dependent radiation corrections.

Such corrections could not - or only at a much reduced scale – be confirmed by empirical data. If they exist they are in any event much smaller for high-frequency data that essentially contribute to the E_S estimate.

Since non of the classical magnitudes , to which both Mw and Me have been scaled, apply source-mechanism corrections, GFZ E_S and Me are calculated also without such.

While scaling $\log M_0$ and thus Mw to long-period Ms(20) is reasonable the scaling of Me to $\log E_S$-Ms(20) means in fact scaling the energy magnitude Me to Mw as a static magnitude reference. This is, however, against Gutenberg's (1956) original intention to relate $\log E_S$ to mB which measures the maximum velocity amplitudes in a wide range of periods that are related to the magnitude- and stress-drop dependent corner period of the radiated source spectrum (see Figs. 1 and 11).

Note that mB_BB scales much better with logEs than Ms(20).

(according to Figs. 4 and 10 OR rms + 0.13 instead of 0.24-0.30!)

A new **Me formula scaled to mB_BB GFZ** would read

$$\mathbf{Me = (logE_S - 1.39)/1.92} \qquad (15)$$

It avoids the systematic underestimation of the energy release by 20 s Ms for $Mw \approx Ms < 7.5$

For $\log E_S \approx 12$ to 17.5, which corresponds to mB_BB ≈ 6 to 8.3, it would yield Me values that are - compared to Ms-scaled Me values - between +0.44 m.u. larger at smaller energies and -0.59 m.u. smallerat the largest energies. Thus (15) would high-light the magnitudes of smaller to moderate earthquakes which radiate relatively more higher and thus damage-relevant frequencies than the very great earthquakes that emphasize on longer period oscillations.

Such an Me would be much more relevant for engineering seismological applications and risk assessment than Mw or an Mw-Ms scaled Me!

Summary

Classical inter-magnitude and magnitude-energy relationships, on which the definition of the currently accepted Mw and Me formulas are based, had not yet been well constrained by data . The same applies to the average Kanamori (1977) E_S/M_0 ratio.

Nevertheless, with much richer modern data and direct instrumental E_S and M_0 determinations being now available, applying more appropriate regressions, and partially significant differences in the defining relationships not withstanding these differences partially cancel each other out when redefining the Mw and Me formulas.

Accordingly, the new Mw formula $Mw = (\log M_0 - 9.11)/1.51$
and the new Me formula (both scaled to Ms!) $Me = (\log E_S - 4.51)/1.51$

are very close to the currently used standard formulas for Mw and Me. The differences in Mw estimates are ≤ 0.07 m.u. and range for Me between -0.10 and -0.14 m.u.

However, this good agreement could only be achieved when GFZ logES data are used which have not been corrected for theoretical source-mechanism dependent radiation efficiency.

When scaling, however, Me not to Ms and thus Mw but to broadband mB measured in a wide range of dominant periods on gets $Me = (\log E_S - 1.39)/1.98.$

Such an Me is likely to be much more appropriate than the Ms-scaled Me or Mw proper for assessing the seismic energy release and thus shaking-damage potential of earthquakes.